Maximum likelihood inversion of simulated LOFAR data

Abhik Ghosh* and Léon V. E. Koopmans
Kapteyn Astronomical Institute, University of Groningen, PO Box 800, 9700 AV Groningen, the Netherlands

Abstract. Accurate knowledge of the foreground emission (both in spatial and frequency domain) is essential to recover the faint EoR signal. In this poster using simulated data of the smooth Galactic Synchrotron foregrounds, we show some initial results of a maximum likelihood (ML) analysis. This method can be applied to LOFAR-EoR data sets before and after foreground removal to know the corresponding maximum likelihood sky.

Keywords: Cosmology: Epoch of Reionization, diffuse radio emission, Methods: Statistical data analysis

1. Introduction

The EoR signal is buried in foreground emission from other astro-physical sources whose contribution is 4 to 5 orders of magnitude larger (Ali et al. 2008). Individual discrete sources can be identified and mostly removed from the images. The contribution from the remaining unresolved sources, diffuse Synchrotron and thermal emission from our Galaxy (Shaver et al. 1999) is still large enough to overwhelm the EoR signal (Di Matteo et al. 2002). Therefore, we need to accurately characterize (and subsequently remove) the diffuse foregrounds for “every image pixel” by taking into account all the direction dependent (DD) and direction independent (DI) effects of the instrument and ionosphere. As a first step it is interesting to see if a regularized ML technique can reproduce these diffuse foreground component over the scales probed by an interferometer and whether the residuals are consistent with the system noise. This poster highlights some initial results of a Maximum Likelihood (ML) inversion methodology based on a Bayesian formalism where no DD and DI errors were included, except for thermal noise.

*email: ghosh@astro.rug.nl
2. A Bayesian approach to model fitting and results

For a linear system like \( \mathbf{d} = \mathbf{f}x + \mathbf{n} \), where \( \mathbf{d} \) is a vector of data points (here visibilities), \( \mathbf{f} \) represents the response function (here Fourier transform kernel), \( x \) is the model parameters that we want to infer given the data and \( \mathbf{n} \) is the noise in the data characterized by the co-variance matrix \( \mathbf{C}_D \), the most likely solution that maximizes the likelihood is

\[
\mathbf{x}_{\text{ML}} = (\mathbf{f}^T \mathbf{C}_D^{-1} \mathbf{f})^{-1} \mathbf{f}^T \mathbf{C}_D^{-1} \mathbf{d}.
\]

In many cases, the problem of finding the most likely solution is ill-posed and we need to introduce priors to regularize the parameter \( x \). Then the most probable solution that maximizes the posterior is

\[
\mathbf{x}_{\text{MP}} = \mathbf{A}^{-1} \mathbf{B} \mathbf{x}_{\text{ML}},
\]

where \( \mathbf{B} = \mathbf{f}^T \mathbf{C}_D^{-1} \mathbf{f} \), \( \mathbf{A} = \mathbf{B} + \lambda \mathbf{C} \), \( \mathbf{C} = \nabla \nabla E_x \). Here, \( \lambda \) is the regularization constant and \( E_x \) is called the regularizing function. Using a foreground model with a power-law power-spectrum and with 22323 LOFAR baselines (within \( \pm 1000 \lambda \) at 150 MHz) on North Celestial Pole (NCP), we reconstructed the MP solution of a confusion limited diffuse foreground slice (Jelić et al. 2008) based on a Bayesian formalism with 2nd order regularization (Fig. 1).

3. Observations and conclusions

We find (and also supported by the log-evidence and reduced chi-square value at the optimal lambda point) that at a resolution of 3.75' for a small patch of sky of around 2° × 2° we can reconstruct the ML sky consistent with the noise level from the simulated data.

References