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On the possibilities of shocks in accretion and wind flows

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Abstract. In the present study, we investigate the possibilities of shocks in the accretion/wind flows. For the constant ultra-relativistic adiabatic index $\gamma(=\frac{4}{3})$, shocks are possible because of the presence of the two saddle type sonic points in the flow. However, employing the relativistic equation of state(EOS) (Synge 1957) in which the adiabatic index varies ($\frac{5}{3} \leq \gamma \leq \frac{4}{3}$) from the non relativistic to relativistic regime with temperature we notice that number of the saddle type sonic point reduces to one (Basu & Mondal 2014, 2011) indicating that the formation of shocks in the flow becomes unlikely (Basu & Mondal 2014; Mondal & Basu 2011).

Keywords : accretion, accretion discs – – shock waves – equation of state – relativity

1. Introduction

In the conventional study of accretion/wind flows people used constant ultra-relativistic (UR) value of $\gamma = \frac{4}{3}$. This particular choice of γ, although takes into account some of the UR-effects, misses the essential feature namely, the variation of γ with temperature. In the case of accretion, matter remains cool (non relativistic(NR)) at the outer boundary and becomes very hot (extreme relativistic) at the inner edge, the situation is reverse in the wind flows. Thus, the variation of γ needs to be taken into account while solving the hydrodynamic equations. However, proper study using the relativistic EOS has not made so far and in the popular studies of numerical relativistic hydrodynamics people prefer to use various alternative models of EOS.

In this work our purpose is to find out the accretion and wind solutions with variable adiabatic index using relativistic EOS. The velocity gradient $\frac{dv}{dr} = \frac{N}{D}$ is derived in a similar way as in the previous study (Chakrabarti 1996a,b; Mondal 2010), however now modified by incorporating this extra piece of information. We then numerically integrate $\frac{dv}{dr}$ to obtain the accretion solutions $v(r)$ for the black hole, neutron star and the wind solutions (Mondal & Basu 2011). Conventionally, these solutions

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are expressed in terms of the Mach no.(ratio of velocity v to sound speed a_s) in order to predict the location of shock in the flow (Chakrabarti 1996b; Mondal 2010). In the present analysis, we follow the same formalism to investigate the possibilities of shock in the flow and find that the solutions have one or two critical points in general and in some cases more (three) (Mondal & Basu 2011).

2. Possibilities of shocks

To find the possibilities of shocks in the flow, we first need to determine the number of critical points in the flow. To form a shock, one requires the presence of two saddle('X')-type (inner and outer) critical points (Chakrabarti 1996a; Mondal 2010). The subsonic matter crossing the sonic point becomes super sonic and may jump to subsonic branch through a shock transition if the flow satisfies shock conditions. These conditions hold in between two 'X'-type sonic points for a large range of parameters specific energy($\mathcal{E}$) and angular momentum(l) values and therefore, a stable or oscillatory shock may form (Chakrabarti 1996a,b). The possibility of occurrence such a shock in the flow is also confirmed numerically (Molteni et al. 1996; Ryu et al. 1997). However, employing relativistic EOS, we re-investigate this issue in the entire range of flow parameters $\mathcal{E}$, l and spin parameter(a) of compact object. In a similar manner of (Chakrabarti 1996b; Mondal 2010), using the sonic point conditions, we express $\mathcal{E}$ as a function of the critical radius r_c. To find the number of critical points in the flow, we plot $\mathcal{E} = \mathcal{E}(r_c)$ in Fig. 1 for the different pairs of (a, l)=(0, 1.3)(top), (0, 1.4)(bottom), (0.5, 1.3)(top), (0.5, 1.33)(bottom), (0.9, 1.299)(top), (0.9, 1.301)(bottom), (0.999, 1.265)(top), (0.999, 1.27)(bottom) values. The Kerr parameter's are printed on the individual curve. For a given (a, l)-curve the number of intersections with a constant $\mathcal{E}$ line (see in figure $\mathcal{E}$=1.00001 line) give the number of critical points in the flow. As we can now see, with the increase of l, in most of the cases curves have two critical points. The situation does not change even if a increases up to $a < 0.9$. In the entire range of $\mathcal{E}$ there is no trace of three critical points. Only one 'X'-type critical point is present. Unfortunately, this is unusual. In Chakrabarti (1996a,b); Mondal (2010) when $\gamma = 4/3$ used, shocks was favourable because a large range of parameters pertained two 'X'-type critical point. In our study, three critical points (among them two are 'X'-type) are rare. We find three critical points just start to occur beyond the $a > 0.9$ values, and become more prominent when $a \rightarrow 1$ (see the curve-(0.999, 1.27) in Fig. 1). However, the value of $\mathcal{E}$ is small and close to one e.g. at $\mathcal{E}$ = 1.00001, curve-(0.999, 1.27) has three critical points: I-inner('X'), M-middle(circle), and O-outer('X'). In-spite of rare possibility, we identify the parameters space $(\mathcal{E}, l)$ at $a = 0.999$, for three critical points. We see in the sub-panel of Fig. 1 that the available range of parameters is small and tiny. This implies a very small window of parameters ($\mathcal{E}$, l & a) can have three critical points in the flow, when temperature dependent adiabatic index is considered. Therefore, possibilities of shocks in the flow which require the presence of two saddle type ('X') critical points becomes very unlikely.

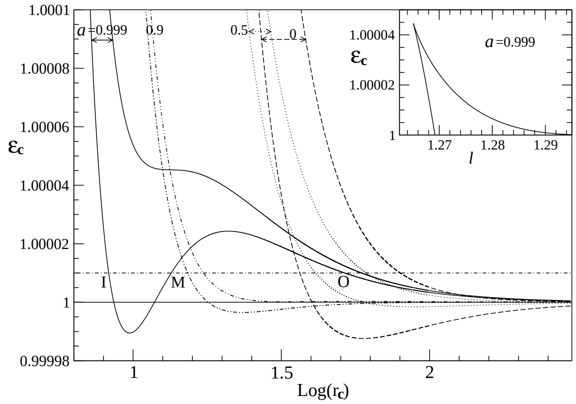

Figure 1. The variation $\mathcal{E}_c$ with r_c is shown for (a, l)=(0, 1.3)(top), (0, 1.4)(bottom), (0.5, 1.3)(top), (0.5, 1.33)(bottom), (0.9, 1.299)(top), (0.9, 1.301)(bottom), (0.999, 1.265)(top), (0.999, 1.27)(bottom with three sonic points). In the sub-panel a parameter space $(\mathcal{E}, l)$ is also drawn when two saddle type ('X') critical points exist.

3. Solution topologies and variation of γ with radius

In the present study, γ is not constant but varies from $\frac{5}{3} \leq \gamma \leq \frac{4}{3}$ from outer to inner boundary. The variation of γ can easily be obtained once we find the solution topologies. We obtain solution topologies by integrating velocity gradient $\frac{dv}{dr}$ (Basu & Mondal 2014; Mondal & Basu 2011) and plot the Mach no. (= $\frac{v}{a_s}$) as a function of r for different inner and outer boundary values. Considering all the different types of solutions (black hole, neutron star and winds), we investigate the variation of γ with the disk radius r. The results are plotted in the Fig. 2. The solutions corresponding to the accretion and wind flows is shown in the sub-panel of the figure. Here the solid curves represent the variation of γ for the accretion flow while the dotted curves represent the variations for the wind solutions. Note that in both the figures up to the critical point (crossing points), γ does not vary much and remain same for both the flows. The variation becomes prominent only after crossing the inner critical point. We also see that in the wind solution the variation of γ is faster than the black hole accretion solution. However, in case of black hole accretion, there is a sharp jump of γ values close to the horizon. Therefore, we conclude that except for the region very nearby the compact object, γ does not change significantly from its NR value. As a result, a wide range of parameters have only one 'X'-type critical points which implies shock is less feasible.

4. Conclusion

In this study, we review hydrodynamic solutions of accretion and wind solutions to investigate the possibilities of shocks in the presence of relativistic EOS (Synge 1957).

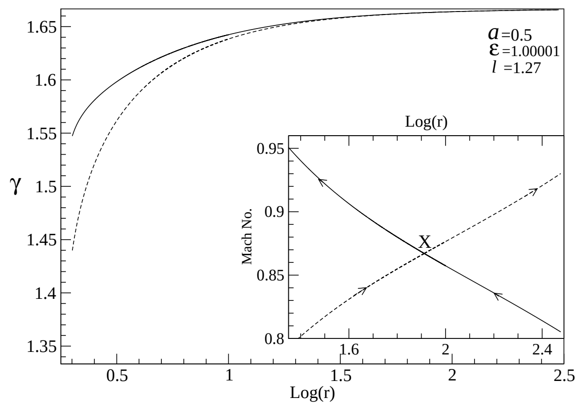

Figure 2. The variation γ as a function of radius is shown in the figure for both accretion(solid) and wind(dotted) flow topologies (in sub-panel).

In the previous study (Chakrabarti 1996a), the flow has mostly two saddle('X')-type sonic points and may contain stable shocks in between two 'X'-type sonic points when the ultra-relativistic $\gamma(=\frac{4}{3})$ is used. However, in the present study with the correct choice of EOS in which γ varies continuously from $\frac{5}{3} \geq \gamma \geq \frac{4}{3}$, we find that the EOS of the matter mostly remains non-relativistic in nature i.e., γ lies close to $\frac{5}{3}$ value. Because of its non-relativistic nature, flow has mostly one saddle type critical points which unfortunately is not favourable for the formation of shock in the flow. These results also agree with our previous similar study (Mondal & Basu 2011).

Editor's Comment: *A difference of opinion between authors and referees is observed in the review process.*

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