Recent Trends in the Study of Compact Objects: Theory and Observation
ASI Conference Series, 2013, Vol. 8, pp 143–146
Edited by Santabrata Das, Anuj Nandi and Indranil Chattopadhyay

Dissipative standing shocks in accretion flows around black holes: a self-consistent analytical study

Biplob Sarkar* and Santabrata Das†
Indian Institute of Technology Guwahati, Guwahati 781039, India

Abstract. We self-consistently study the properties of the dissipative standing shock waves in an accrertion flow around a stationary black hole. We use analytical method to achieve our goal and identify an effective area in the parameter space spanned by the specific energy and the specific angular momentum which allows accretion flow to pass through shock having some energy dissipation. As the dissipation is increased, the parameter space is reduced and finally disappears when the dissipation is reached its critical value. We show the variation of shock location and compression ratio as a function of the specific energy and study them in terms of energy dissipation across the shock.

Keywords : black hole physics – accretion, accretion discs – shock waves – hydrodynamics

1. Introduction

The accretion flow around the black holes must be transonic in nature in order to satisfy the inner boundary conditions. Moreover, low angular momentum accretion flow may possess multiple sonic points and pass through the shock waves depending on the outer boundary conditions (Chakrabarti 1989; Chakrabarti 1996; Das et al. 2001). For a thin, axisymmetric, adiabatic accretion flow around a Schwarzschild black hole, Das et al. (2001) developed a completely analytical method to study the properties of sonic points and shock waves. In this work, we consider the shocks are dissipative in nature where a part of the accreting energy is radiated away via the disk surface at the shock location and accordingly, we modify the Rankine-Hugoniot shock conditions to calculate the standing shock properties.

*biplob@iitg.ernet.in
†sbdas@iitg.ernet.in

2. Model equations

We consider thin, inviscid and steady accretion flow around a Schwarzschild black hole. We assume that the flow is in hydrostatic equilibrium in the transverse direction and the space-time geometry around the black hole is described by the Paczyński-Wiita pseudo-Newtonian potential $\Phi = \frac{1}{2(x-1)}$ (Paczyński & Wiita 1980), where, x is the radial distance from the black hole measured in unit of Schwarzschild radius ($r_g = 2GM_{BH}/c^2$). Here, M_{BH}, G and c are the mass of the black hole, the gravitational constant and the velocity of light, respectively. Accordingly, time and mass is measured in unit of r_g/c and M_{BH}. With this, the dimensionless energy conservation equation (Chakrabarti 1989) is given by,

$$\mathcal{E} = \frac{v^2}{2} + \frac{a^2}{\gamma - 1} + \frac{\lambda^2}{2x^2} - \frac{1}{2(x-1)}, \tag{1}$$

where, v, a, $\mathcal{E}$ and λ are the velocity, sound speed, specific energy and specific angular momentum of the flow, respectively. Here, $n(=\frac{1}{\gamma-1})$ is the polytropic constant and γ is the adiabatic index. The mass flux conservation equation (Chakrabarti 1989) is given by,

$$\dot{\mathcal{M}} = v\, a^q\, x^{3/2}(x-1), \tag{2}$$

where, $q = (\gamma+1)/(\gamma-1)$. Here, we use $a^2 = \gamma P/\rho$, where, P and ρ are the isotropic pressure and density of the flow at the disk equatorial plane.

Following the sonic point analysis (Chakrabarti 1989), one obtains algebraic equation for sonic point (Das et al. 2001) as,

$$\mathcal{N} x_c^4 - \mathcal{O} x_c^3 + \mathcal{P} x_c^2 - \mathcal{Q} x_c + \mathcal{R} = 0 \tag{3}$$

where, $\mathcal{N} = 10\mathcal{E}$, $\mathcal{O} = 16\mathcal{E} + 2n - 3$, $\mathcal{P} = 6\mathcal{E} + \lambda^2(4n-1) - 3$, $\mathcal{Q} = 8n\lambda^2$, and $\mathcal{R} = (1+4n)\lambda^2$. Eq. (3) is a quartic equation having four roots. Among them, all the roots may be complex or two complex and two real or all four are real (Abramowitz and Stegun 1970). A necessary condition to form a shock wave is to have four real roots (Das et al. 2001). We observed that one root remains inside the horizon and others lie outside and out of them two would be X-type and one in between must be O-type.

3. Shock location analysis

For standing shock, Rankine-Hugoniot (R-H) conditions need to be satisfied (Landau & Lifshitz 1959; Das et al. 2010). Since the radiative dissipation is expected mostly through thermal Comptonization (Chakrabarti and Titarchuk 1989), the energy balance condition across the shock is modified as $\mathcal{E}_+ = \mathcal{E}_- - \Delta\mathcal{E}$, where $\Delta\mathcal{E} = fn(a_+^2 - a_-^2)$. Here, '-' and '+' denote pre- and post-shock quantities and the parameter f is the fraction of difference in thermal energy dissipated across the shock which is a measure of the energy dissipation. Using R-H conditions along with Eqs. (1-2), we obtain the

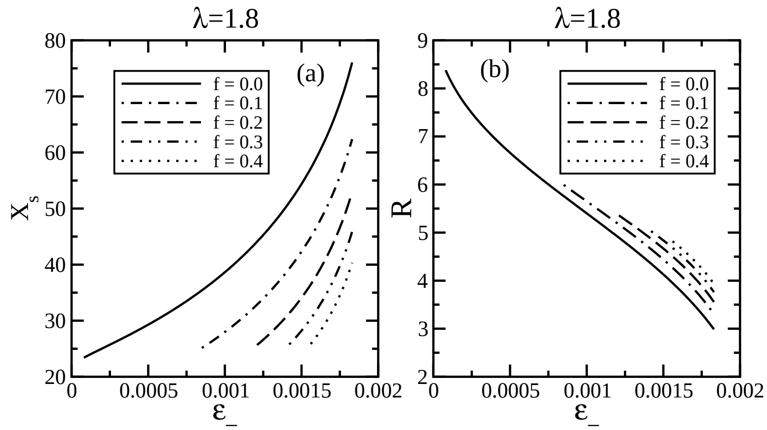

Figure 1. (a) Shock location x_s as a function of pre-shock energy $\mathcal{E}_-$. (b) Variation of compression ratio R with pre-shock energy $\mathcal{E}_-$. In both the graphs the solid curve represents the shock locations without any energy dissipation.

shock invariant quantity in terms of pre- and post-shock Mach numbers (M_- and M_+) as,

$$C = \frac{[M_+(3\gamma-1)+(2/M_+)]^2}{2+(\gamma-1)M_+^2+2f} = \frac{[M_-(3\gamma-1)+(2/M_-)]^2}{2+(\gamma-1)M_-^2+2f}, \tag{4}$$

Simplifying Eq. (4), we obtain,

$$2(\gamma-1)(M_+^2+M_-^2)-[(3\gamma-1)^2-2(3\gamma-1)(\gamma-1)+f(3\gamma-1)^2]M_+^2M_-^2+4+4f = 0. \tag{5}$$

Following Das et al. (2001), we expand the pre- and post-shock Mach number as quadratic polynomial of x and insert them in Eq. (5) to obtain,

$$\mathcal{A}x_s^4 + \mathcal{B}x_s^3 + \mathcal{C}x_s^2 + \mathcal{D}x_s + \mathcal{F} = 0. \tag{6}$$

Here, we use $\gamma = 4/3$ for relativistic flow and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{F}$ are the functions of the flow parameters $\mathcal{E}$ and λ (Das et al. 2001). Eq. (6) is quartic in nature which we solve analytically.

In Fig. 1a, we show the variation of shock location (x_s), same as x_{s3} in Chakrabarti (1989), as function of the pre-shock energy ($\mathcal{E}_-$) for $\lambda = 1.8$. Curves plotted with different line styles are for various energy dissipation at the shock marked in the sub-panel. At zero dissipation ($f = 0$), shocks form for a wide range of energy. As the dissipation is increased, shocks move towards the black hole and the available range of energy for shock is shortened. Finally, shocks disappear completely when dissipation reached its critical value. In Fig. 1b, we depict the variation of compression ratio ($R = v_-/v_+$) corresponding to Fig. 1a. We find that R increases with energy dissipation for a given $\mathcal{E}_-$. This is expected as the shift of shock location towards the black hole ensures further compression of post-shock matter enhancing the compression ratio.

As anticipated above, the possibility of shock formation is reduced as we increase

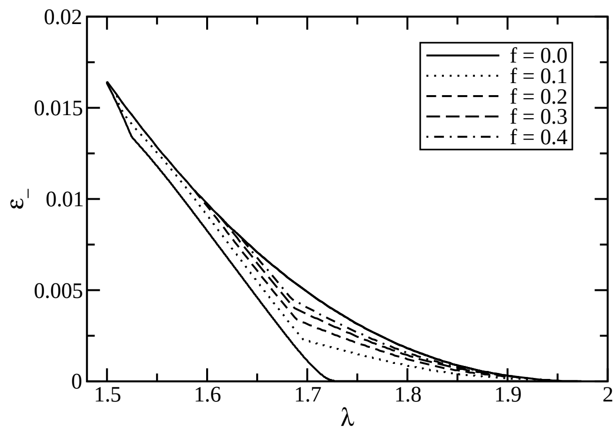

Figure 2. Variation of the parameter space responsible for the formation of a standing shock as a result of energy loss $\Delta\mathcal{E}$. See text for details.

the dissipation. In order to understand this, we identify the region of the parameter space (spanned by the specific energy and specific angular momentum of the flow) for shock which is shown in Fig. 2. Solid boundary represents the zero dissipation limit ($f = 0$). With increasing energy dissipation identified with different line styles parameter space shrinks and eventually disappears.

4. Conclusion

In this work, we demonstrate that the shock location and its properties could be studied completely analytically even in the presence of energy dissipation across the shock. The radiative loss removes a significant thermal pressure from the inflow and as the dissipation increases, the shock location shifts towards the black hole to maintain the pressure balance across it. We also find that the parameter space for standing shocks decreases with the increase of energy dissipation and vanishes completely when the dissipation is reached its critical value.

References

Abramowitz M., Stegun, I. A., 1970, Handbook of mathematical functions, Dover Publications, INC., New York

Chakrabarti S. K., 1989, ApJ, 347, 365

Chakrabarti S. K., Titarchuk L. G., 1995, ApJ, 455, 623

Chakrabarti S. K., 1996, ApJ, 464, 664

Das S., Chattopadhyay I., Chakrabarti S. K., 2001, ApJ, 557, 983

Das S., Chakrabarti S. K., Mondal S., 2010, MNRAS, 401, 2053

Landau L. D., Lifshitz E. D., 1959, Fluid Mechanics, New York: Pergamon

Paczyński B., Wiita P., 1980, A&A 88, 23