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Emergent perspective of cosmology and the solution to the cosmological constant problem

T. Padmanabhan*
Inter University Centre for Astronomy & Astrophysics, Pune 411 007, India

Abstract. The emergent gravity paradigm allows one to view the expansion of the universe as a quest for holographic equipartition. This perspective suggests introducing a new, dimensionless, conserved number N_c ('CosMIn') which counts the number of modes crossing the Hubble radius during the three phases of evolution of the universe, viz., the inflationary phase, the matter/radiation dominated phase and the late time acceleration phase. Theoretical considerations suggest that $N_c \approx 4\pi$. *This single postulate leads us to the correct, observed, numerical value of the cosmological constant!* This approach also provides a unified picture of cosmic evolution relating the early inflationary phase to the late accelerating phase.

Keywords : cosmology: theory, cosmological parameters, early universe

1. Strangeness of our universe

The currently successful standard model of cosmology leads to a theoretically disjoint and ad-hoc description of the universe. Among the issues which could be conceptually bothersome are the following questions, which are not even addressed within the conventional perspective of cosmology. (For a discussion of the first two issues, see Padmanabhan (2010a); for a discussion of last two, see Padmanabhan (2012b); Padmanabhan & Padmanabhan (2013)).

- Why is our universe described by a solution to Einstein's equation which is maximally symmetric and dynamically expanding? Usually, a solution is selected by initial/boundary conditions but the classical singularity prevents us from imposing an *initial* conditions for our universe.

*email: paddy@iucaa.ernet.in

- How come the universe made a spontaneous transition from a quantum mechanical description to a classical description? Laboratory quantum systems do not spontaneously become classical under closed Hamiltonian evolution.
- The universe at large scales breaks Lorentz invariance through the existence of a Lorentz frame in which it is isotropic. For example, CMBR observations routinely allow us to determine our absolute velocity with respect to the cosmic ether.
- The evolution of our universe is described by three separate phases (the inflationary phase, the matter/radiation dominated phase and the late time acceleration phase) with no apparent connection with each other! Each of these phases are described by an arbitrary set of parameters essentially thrusted upon us from observations.

These considerations suggest that we may be wrong in studying cosmology as a special solution to gravitational field equation. It is probably necessary to approach cosmology from a more fundamental principle which, of course, should have close ties with the description of gravity itself.

The emergent gravity paradigm (for a review, see Padmanabhan, 2010b), which attributes to gravitational field equations the same status as equations of elasticity or fluid mechanics, provides a viable candidate for such an alternative description. Recent work based on this paradigm (Padmanabhan, 2012b) has shown that cosmic evolution can be thought of as a quest for *holographic equilibrium* in the following sense. One can associate with the *proper* Hubble volume of the universe, $V_{prop} \equiv 4\pi/3H^3$, the numbers,

$$N_{sur} \equiv 4\pi H^{-2}/L_P^2; \qquad N_{bulk} \equiv -\epsilon E/(1/2)k_B T \tag{1}$$

which count the surface and bulk degrees of freedom, where $E = (\rho + 3p)V_{prop}$ is the Komar energy, $T = H/2\pi$ is the analogue of the horizon temperature and $\epsilon = \pm 1$ is chosen to keep N_{bulk} positive. Clearly, $|E| = (1/2)N_{bulk}k_B T$ denotes equipartition of energy. Holographic equipartition is the demand that $N_{sur} = N_{bulk}$, which holds in a de Sitter universe with $p = -\rho$, $\epsilon = 1$, $H^2 = (8\pi/3)\rho$. When the universe is not pure de Sitter, we expect the holographic discrepancy between N_{sur} and N_{bulk} to drive the expansion of the universe, which suggests (Padmanabhan, 2012a) the law:

$$\frac{dV_{prop}}{dt} = L_P^2(N_{sur} - \epsilon N_{bulk}) \tag{2}$$

Incredibly, this leads to the standard Friedmann equation for cosmic expansion, but now obtained without *using the field equations of general relativity!* The right hand side is (nearly) zero in the initial (inflationary) and final (accelerated) phases in the evolution of the universe (with $V_{prop} \approx$ constant) and cosmic expansion in the transient phase can be interpreted as a quest towards holographic equipartition.

While such an approach provides a fresh perspective on cosmic evolution, we would like to do more using this paradigm. In particular, if the three phases in the evolution of the universe are described by the above equation, then it should be possible to obtain some quantity which remains constant when the universe passes through the three phases. The numerical value of such an invariant constant — which is common to the three phases — will then provide a simple way of relating the Hubble radius at the final phase (and thus the cosmological constant) with the Hubble radius during the inflationary phase. We will now show that, incredibly enough, such a conserved dimensionless number exists and it can be used to determine the numerical value of the cosmological constant.

2. CosMIn and the cosmological constant

A proper length scale $\lambda_{\rm prop}(a) \equiv a/k$ (labelled by a co-moving wave number, k) crosses the Hubble radius whenever the equation $\lambda_{\rm prop}(a) = H^{-1}(a)$, i.e., $k = aH(a)$ is satisfied. For a *generic* mode (see figure 1; line marked ABC), this equation has three solutions: $a = a_A$ (during the inflationary phase; at A), $a = a_B$ (during the radiation/matter dominated phase; at B), $a = a_C$ (during the late-time accelerating phase; at C). But modes with $k < k_-$ exit during the inflationary phase and *never* re-enter. Similarly, modes with $k > k_+$ remain inside the Hubble radius and only exit during the late-time acceleration phase.

The modes with comoving wavenumbers in the range $(k, k+dk)$ where $k = aH(a)$ and $dk = [d(aH)/da]da$ cross the Hubble radius during the interval $(a, a + da)$. The number of modes in a comoving Hubble volume $V_{\rm com} = (4\pi H^{-3}/3a^3)$ with wave numbers in the interval $(k, k + dk)$ is $dN = V_{\rm com} d^3k/(2\pi)^3$. Hence, the number of modes that cross the Hubble radius in the interval $(a_1 < a < a_2)$ is given by

$$N(a_1, a_2) = \int_{a_1}^{a_2} \frac{V_{\rm com} k^2}{2\pi^2} \frac{dk}{da}\, da = \frac{2}{3\pi} \int_{a_1}^{a_2} \frac{d(Ha)}{Ha} = \frac{2}{3\pi} \ln\left(\frac{H_2 a_2}{H_1 a_1}\right), \qquad (3)$$

where we have used $V_{\rm com} = 4\pi/3H^3a^3$ and $k = Ha$.

All the modes which exit the Hubble radius during $a_A < a < a_X$ enter the Hubble radius during $a_X < a < a_B$ (and again exit during $a_Y < a < a_Q$.) So the number of modes which do this during PX, XY or YQ is a *characteristic, 'conserved' number ("CosMIn")* for our universe, say N_c. Its importance is related to the the cosmic parallelogram $PXQY$ (figure 1) which arises *only* in a universe having three distinct phases (Bjorken, 2004; Padmanabhan, 2008; Padmanabhan, 2012a). As shown in figure 1, these modes in $PXQY$ (with $k_- < k < k_+$) cross the Planck length during $a_- < a < a_+$. Based on holographic considerations, it is possible to argue (Padmanabhan, 2012b) that Planck scale physics imposes the condition $N_c = N(a_-, a_+) \approx 4\pi$ at this stage. So, by computing CosMIn for the universe, and equating to 4π, we can determine ΛL_P^2. A simple calculation now shows (Padmanabhan & Padmanabhan,

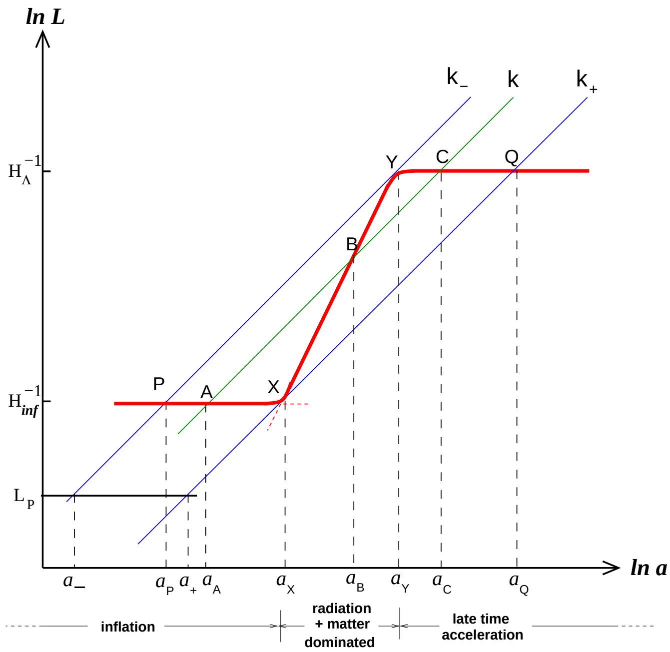

Figure 1. Various length scales in the universe; see text for description.

2013) that

$$\Lambda L_P^2 = \beta^{-2} C(\sigma) \exp[-24\pi^2 \mu]; \qquad \mu \equiv \frac{N_c}{4\pi} \tag{4}$$

where $\beta^{-1} \equiv H_{\rm inf} L_P$, $\sigma^4 \equiv (\Omega_R^3/\Omega_m^4)[1 - \Omega_m - \Omega_R]$ and $C(\sigma) = 12(\sigma r)^4 (3r + 4)^{-2}$ where r satisfies the quartic equation $\sigma^4 r^4 = (1/2)r + 1$. Given the numerical value of σ, the inflation scale determined by β, and our postulate $\mu = 1$, we can calculate the value of ΛL_P^2 from Eq. (4).

The result in Eq. (4) is summarized in figure 2. The thick black curve is obtained from Eq. (4) if we take $\mu = 1$ and $\beta = 3.83 \times 10^7$ (corresponding to the inflationary energy scale of $V_{\rm inf}^{1/4} = 1.16 \times 10^{15}$ GeV) and leads to the observed (mean) value of $\Lambda L_P^2 = 3.39 \times 10^{-122}$ (horizontal unbroken, blue line). Observational constraints lead to $\sigma = 0.003^{+0.004}_{-0.001}$ (three vertical, red lines) and $\Lambda L_P^2 = (3.03 - 3.77) \times 10^{-122}$ (horizontal, broken blue lines). This cosmologically allowed range in σ and ΛL_P^2 is bracketed by the two broken black curves obtained by varying β in the range $(2.64 - 7.29) \times 10^7$ (i.e, $V_{\rm inf}^{1/4} = (0.84 - 1.4) \times 10^{15}$ GeV). So, for an acceptable range of energy scales of inflation, and for the range of σ allowed by cosmological observations, our postulate $N_c = 4\pi$ gives the correct value for the cosmological constant.

Since our results only depend on the combination $\beta^{-2} \exp(-24\pi^2\mu)$, the same set of curves arise in a Planck scale inflationary model ($\beta = 1$) with μ in the range

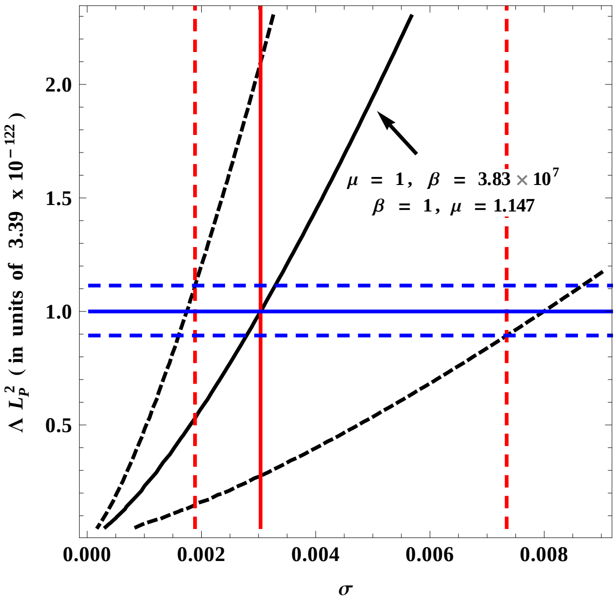

Figure 2. Determination of ΛL_P^2; see text for discussion.

(1.144 – 1.153). There are three conceptually attractive features about Planck scale inflation with $\beta = 1$. First, it eliminates one free parameter, β, and gives a direct relation between the two scales Λ and L_P^2 which occur in the Einstein-Hilbert action. (The dependence of the result on σ is weak and can be thought of as a matter of detail, like, for example, the fine structure correction to spectral lines beyond Bohr's model). Second, we can think of the intermediate phase as a mere transient connecting two de Sitter phases (the chicken is just the egg's way of making another egg!), both of which are semi-eternal. Since the de Sitter universe is time-translationally invariant, it is a natural candidate to describe the geometry of the universe dominated by a single length scale — L_P in the initial phase and $\Lambda^{-1/2}$ in the final phase. The quantum instability of the de Sitter phase at the Planck scale can lead to cosmogenesis and the transient radiation/matter dominated phase, which gives way, eventually, to the late-time acceleration phase. Finally, the argument for $N_c \approx 4\pi$ is quite natural with Planck scale inflation. The transition at X, entrenched in Planck scale physics in such a model, can easily account for deviations of μ from unity.

Solving the cosmological constant problem *by actually determining its numerical value* has *not* been attempted before. (For a classification of approaches to the cosmological constant problem, see Nobbenhuis, 2006). This approach is similar in spirit to the Bohr model of the hydrogen atom, which used the postulate $J = n\hbar$ to explain the hydrogen spectrum. Here, our postulate $N_c = 4\pi$, captures the essence and explains the value of ΛL_P^2. This is simpler and more elegant than many other ad-hoc assumptions made in the literature (for a recent review, see Martin, 2012) to solve the cosmological constant problem.

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