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# Aperture synthesis in gravitational wave search

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**Abstract.** Global network of sensitive advanced interferometric broad band gravitational wave (GW) detectors will finish upgrades soon. The projected distance reach for aLIGO, aVIRGO and KAGRA is few hundred mega-parsec (few tens of mega-parsec for iLIGO) to observe compact binaries with neutron stars and black holes. GW search carried out in a coherent multi-detector mode would enhance this distance further. In this mode, the data is phase coherently combined from all the interferometers; mimicking the aperture synthesis in the GW search. Here, we review the aperture synthesis technique in GW context listing the advantages in terms of event rates, sky coverage and angular resolution.

*Keywords* : gravitational waves – aperture synthesis – compact binaries

## 1. Electromagnetic vs gravitational wave window

Modern astronomy aided with sophisticated telescopes has enhanced our understanding of the universe progressively. It provides a multi-wavelength picture in Electromagnetic (EM) window (radio waves (10 kHz) to gamma rays ($10^{19}$ Hz)) in analogy to the colourful world we see around us. The intensity and features of the universe vary in different frequencies due to the difference in underlying radiative process. Gamma ray bursts (GRB) emit strong gamma rays due to nuclear reactions. The x-rays emitted from the accreting binaries are due to the electronic transitions. Similarly the atomic and molecular transitions emit infra-red radiation from molecular clouds having hot stars. Pulsars emit radio waves due to associated synchrotron radiation from motion of relativistic electrons giving beaming effect. Thus, pertaining to a large class of radiative processes, the universe appears *multicoloured* (emitting different strengths) when *viewed* with the EM periscope.

Gravitational waves (GW); prediction of Einstein's General Theory of Relativity,

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are oscillations of space-time fabric analogous to EM waves being oscillations of EM fields. The leading order of EM radiation is dipolar whereas that of GW is quadrupolar in nature. Thus, physical system with non-zero non-spherical kinetic energy produces GWs which act as perturbations in the space-time fabric propagating with speed of light. The dimensionless GW amplitude scales as (Thorne 1980)

$$h \sim \frac{4G\, K_{non-spherical}}{r\, c^4} \tag{1}$$

where $r$ is the distance to the source and $K_{non-spherical}$ is the non-spherical part of the kinetic energy [1]. Unlike the EM window, GW window probes the macroscopic dynamics of the astrophysical system which is governed by the self-gravity. The GW angular frequency of this self-gravitating system is its dynamical frequency $\omega \sim \sqrt{\pi G\, \rho}$ (Sathyaprakash & Schutz 2009) Thus,

$$f_{GW} = 2kHz \left[\frac{M}{1.4M_{\odot}}\right]^{1/2} \left[\frac{R}{10km}\right]^{-3/2} . \tag{2}$$

Referring to Eq. (2), $f_{Sun} \sim 0.1$ mHz and $f_{NS} \sim 2$ kHz are the GW frequencies[2] of potential GW sources for space based (eLISA) and ground based detectors (aLIGO)[3]. It is worth noticing that this gives 7 orders of magnitude difference in frequency band. Thus, together aLIGO and eLISA would cover frequency range from a few mHz to a few kHz spanning a variety of astrophysical sources which include inspiralling compact binaries, pulsars, SN bursts, GRBs, super-massive black holes (SMBHs), intermediate mass BHs (IMBH) and stochastic GW background. The GWs emitted from these sources interact weakly with matter and hence their detection is a technological challenge in itself. On the flip-side, they do not undergo any scattering/ absorption before reaching the detector. Further, the frequency band of astrophysical GWs falls in the audible range which would allow us to *listen* to the symphony of the universe as opposed to *view* the multi-wavelength universe.

## 2. Network of ground based interferometric GW detectors

The km arm-length broad-band (few tens Hz -1 kHz) ground based GW interferometric detectors measure the phase difference due to the the strain produced in the arm-length during the passage of the GWs. The existing km arm-length initial level projects LIGOs (US), VIRGO (French-Italian) have taken data during the first science run S1 with the sensitivity $h \sim 10^{-17}$ which improved to $h \sim 10^{-23}$ in the S5 run. During S5 run (2007), joint data was taken and analyzed along-with the VSR1 (VIRGO

[1] A terrestrial GW source produces negligible GW amplitude; e.g. 10 m solid rod with mass of 1 ton rotating at 10 Hz on Earth would produce $h \sim 10^{-43}$ on other side of the globe

[2] Here $R$ is taken as a typical size of the GW source. In case of binaries, it is the orbital radius and for isolated object, it is the radius of the object.

[3] aLIGO – advanced LIGO, iLIGO – initial LIGO

science run). As yet, there is no direct detection of GW. Currently, the detectors are undergoing upgrades to improve the sensitivity level enabling them to probe deeper into the universe. The average distance reach of the iLIGO detectors during the S5 run was 20 Mpc (reaching the VIRGO cluster) for the compact inspiralling binaries. Based on the population synthesis simulation models, the typical detection rates per year for binary neutron stars (NS) are $2 \times 10^{-4}(0.2)$, for NS-BH are $7 \times 10^{-5}(0.1)$ and for binary BH are $2 \times 10^{-4}(0.5)$ (Abadie et al. 2010)[4]. The aLIGO is designed to improve the strain sensitivity by a factor 10 over the iLIGO. As the detector measures $h \propto 1/r$ and not the power (unlike any EM detector), this would improve the distance reach of the detector and hence the events by a factor of 1000 making GW detection a reality. Further, 3 km arm-length Kamioka GW detector (KAGRA) is under construction in the Kamioka mine in Japan and a proposal of LIGO-India [5] is under review.

## 3. Aperture synthesis in astronomy

Over the last century, advanced terrestrial as well as space telescopes have given a *deeper, brighter and resolved* view of the universe in the EM window. For example, to achieve this, in optical/radio band, larger telescopes were built to observe fainter as well as clearly resolved objects ($\theta \sim \lambda/D$; D is the diameter of the primary objective). The brightness can be further enhanced by source tracking. However, there is a technological limitation in constructing bigger telescopes and tracking with them. The technique of Aperture synthesis uses the concept of multi-slit interferometry to improve the resolution. It combines the signal from different telescopes to construct an effective telescope offline which gives better resolution and in turn mimics an effective telescope with larger aperture. It is used in optical, infrared and radio astronomy. To give an example, the VLA (Very Large Array) has 27 component radio telescopes (25 m diameter) with a maximum baseline of 36 km which gives the smallest angular resolution of 0.05 arc-seconds at 7 mm. Similar technique is used in VLT (Very Large Telescope), VLBI (Very Large Baseline Interferometry).

## 4. Aperture synthesis in GW inspiral search

Below, we list some of the features involved in the GW search:

**Directional Response** – Directional response of each GW interferometer is broad and covers one third of the sky (see Fig. 1) in contrast to the beamed directional response of the EM antennas. As a result, the data stream would contain GW signals from large fraction of the sky which makes the GW data mining a unique exercise by itself. In

[4]The numbers in outside(inside) the bracket are the pessimistic(optimistic) rates.

[5]One of the 3 LIGO detectors to be shipped in the Indian subcontinent

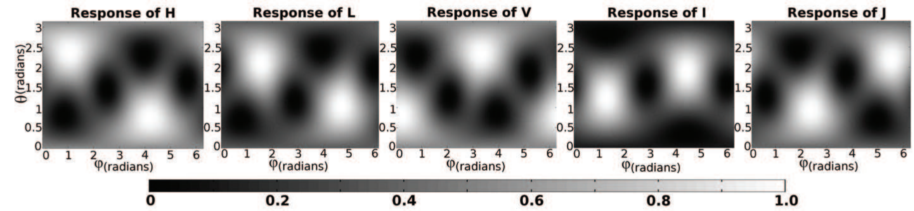


**Figure 1.** Antenna response $(F_+^2 + F_x^2)$ of LIGO-Hanford(H), LIGO-Livingston(L), VIRGO(V), LIGO-India(I), KAGRA(J).

addition, only handful of sparsely located detectors are enough to cover the entire sky.

**Source tracking** — The source tracking of a continuous GW source (e.g. Pulsar) is carried out by the software incorporating the Doppler modulation due to the detector's motion in contract to the physical motion of the telescope.

**Source location** — The data contains signals from all the directions and hence using the triangulation method incorporating the delays, the source can be located.

**Matched filtering** — Detecting modeled signal in noisy data requires special phase matching techniques, namely matched filtering. The templates are constructed for the entire physical parameter space of modeled signals (*e.g.* GW from Pulsars or inspiraling binaries) with which the data is cross-correlated and a maximum is chosen.

In literature, several authors have addressed the GW detection with a multi-detector network for detecting GW short bursts of few msec duration (Klimenko et al. 2005; Pai et al. 2008) as well as from the inspiralling binaries (Pai et al. 2001; Finn 2001; Harry & Fairhurst 2011). Here, I present the numbers for the inspiralling binary sources with NS and BHs as components with mass $< 10M_\odot$ in the multi-detector context as they are one of the prominent sources for aLIGO which acquire enough SNR during the inspiral phase. The accurate modeling of the inspiral phase using post-Newtonian theory enables us to accurately determine the chirp mass $\mathcal{M}$ via matched filtering. Since the inspiral signal $h \sim \mathcal{M}^{5/3}/r$, determination of chirp mass helps us to determine the distance to the source *via* the signal-to-noise ratio (SNR). The GW detection of inspiral phase with multiple interferometers would determine the inclination of the binary (otherwise not accessible with the EM observations) (Sathyaprakash & Schutz 2009).
The GW from the inspiralling compact binary is given by

$$\mathbf{s} = F_+\mathbf{h}_+ + F_x\mathbf{h_x} \tag{3}$$

where $F_+, F_x$ are the antenna pattern functions which depict the directional response of the interferometer to the two GW polarizations $+, \times$. A network signal $\mathbf{S} = [\mathbf{s_1}, \mathbf{s_2}, ..., \mathbf{s_d}]$ can be constructed from the time series of the **d** individual detectors. Using the singular value decomposition, **S** can be decomposed into two individual effective synthetic detectors each of which are constructed from the phase coherent

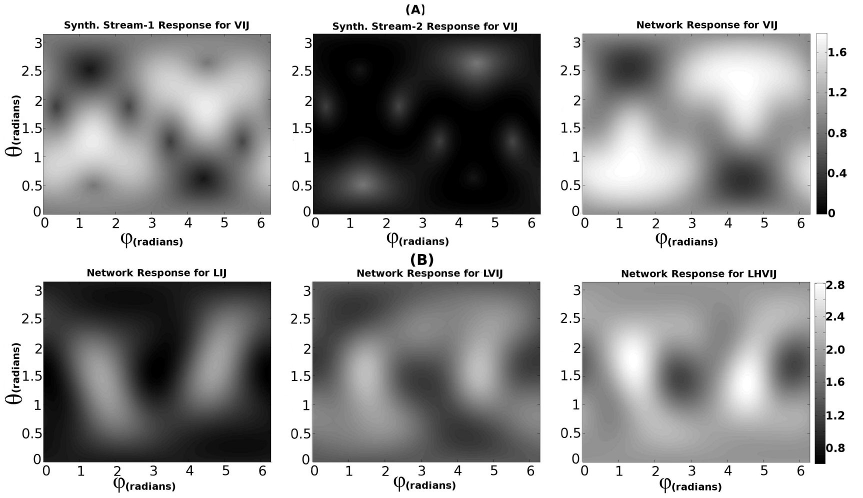


**Figure 2.** (A) VIJ network – Antenna pattern of the synthetic detectors and the whole network. (B) Improved network antenna pattern with detector networks LIJ, LVIJ, LHIVJ

linear combination of the individual data streams assuming the white, Gaussian noise and pertaining to a particular source location as $\sum_I \hat{F}^I_{+,\times} s_I$ (Harry & Fairhurst 2011)[6,7]. In Fig. 2, we show the antenna pattern response of the synthetic detectors as well as the whole network for various detector networks. For face-on binaries, this would be proportional to $SNR^2$ of the synthetic stream/network respectively.

Using Figs 1 and 2-B, the improvement in the event rate via the distance reach (Schutz 2011) (for face-on binaries) computed with a 3-detector network is 5.5 fold, a 4-detector network is 8-fold and a 5-detector network is 12-fold as compared to a single LIGO like detector. Thus, these numbers are over and above the event rates given for the advanced detectors in section 2. Further, a recent study of the angular resolution (Fairhurst 2011) suggests that for 4-detector site network, majority inspiral signals would be localized within 20 sq. degrees with 20% within 5 sq. degrees. With 5 detector site, all the signals would be within 20 sq. degrees.

## 5. Conclusion

An outline of the aperture synthesis technique with its advantages such as event rates, angular resolution etc in the GW detection context is reviewed. Though advantageous, all sky search using matched filter technique with the aperture synthesis is computa-

[6] Haris M. K. and A. Pai, in preparation.

[7] This is expressed in the Dominant Polarization frame with a specific choice of $\psi$ given in (Harry & Fairhurst 2011). The I corresponds to I-th detector

tionally expensive (Pai et al. 2001). Low latency sky search technique is one of the open questions in the field.

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